NONSTATIONARY PROCESS OF HEAT TRANSFER IN A TUBE WITH LONGITUDINAL FINS

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An analytical solution to the problem of nonstationary thermal interaction of a flow of a heat-transfer agent and a thin-walled tube with longitudinal fins is constructed for variable parameters of heat transfer.

Heat-exchange devices that include a thin-walled tube with longitudinal fins as a component find application in modern power engineering, chemical engineering, cryogenic engineering, and a number of other branches of the national economy.

A linear model of a nonstationary process of heat transfer in the device described is investigated in [1] using a one-sided integral Laplace transform in the time variable. The conditions for suitability of the method made it impossible to consider the case of variable parameters of heat transfer, which is of interest for practical applications. The indicated difficulty was overcome in [2] by using a finite-difference scheme of run, modified for a complex multinodal graph. In recent calculations of nonstationary temperature fields in elements of heat-exchange equipment and power-generating devices good use is made of the method of the integral Laguerre transform in the time variable, which makes it possible to take account of a change in the transfer parameters along the spatial coordinate and with time within the framework of a linear approximation [3–6].

If well-known limitations are met, the integral Laguerre transform [7] assigns to each function $f(t)$ a column vector f whose components are defined by the rule

$$f_n = (f\,;\,L_n)\,,\quad n = 1\,,\,2\,,\,3\,,\,\ldots, \tag{1}$$

where

$$L_n(t) = \frac{\exp t}{n\,!}\,\frac{d^n}{dt^n}\,[\exp(-t)\,t^n]\,;$$

$$(x\,,\,y) = \int_0^\infty x(t)\,y(t)\exp(-t)\,dt\,.$$

The inverse transform has the form

$$f(t) = \mathbf{f}^{\mathrm{T}}\cdot\mathbf{L}(t) = \sum_{n=0}^{\infty} f_n L_n(t)\,, \tag{2}$$

where T denotes transposition and the dot denotes scalar multiplication of vectors. Below, substantial use is made of a functional property of the integral Laguerre transform:

$$\left(\frac{df}{dt}\,;\,L_n\right) = \sum_{k=0}^{n} f_k - f(0)\,, \tag{3}$$

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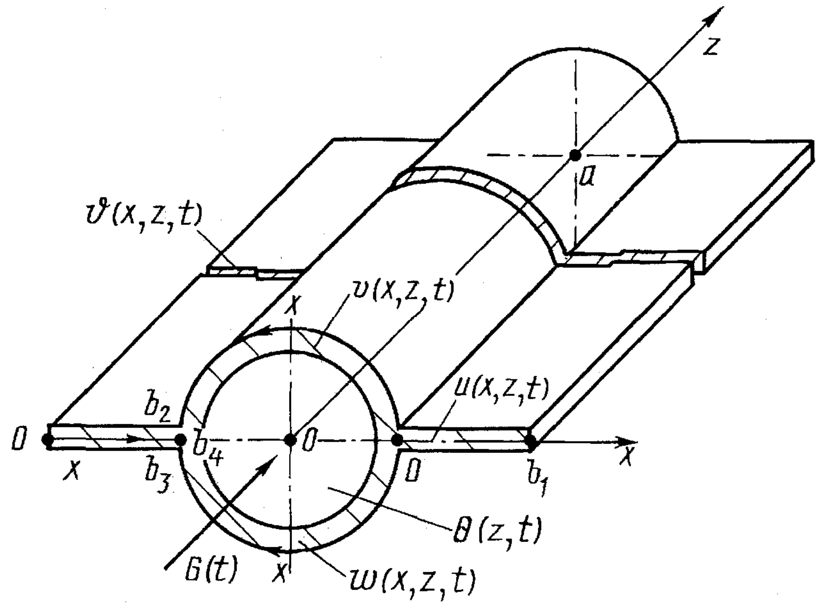

Fig. 1. Scheme of the process of heat transfer in a longitudinally finned tube.

which, in symbolic form appears as

$$\frac{df}{dt} \Rightarrow Df - f_0\,;$$

where D is an infinite dimensional matrix whose elements above the diagonal are zero and below the diagonal and on the diagonal are unity, and the components of the column vector f_0 are the values of $f(0)$.

We consider the nonstationary process of conjugate heat transfer in a thin-walled tube with longitudinal fins in its interaction with a flow of a heat-transfer agent whose scheme is shown in Fig. 1. Let all thermophysical characteristics of the flow and the tube and fin material be independent of the temperature, the coefficient of heat transfer from the heat-transfer agent to the tube wall be independent of the heat-transfer agent and wall temperatures, the temperature of the flow of the heat-transfer agent be constant over the cross section and depend only on the time t and the longitudinal coordinate z, and the tube wall and fin temperatures be constant over the thickness and depend on the time t, the longitudinal coordinate z, and the transverse coordinate x.

Below we assume that the heat-transfer coefficients are known functions of the coordinates and the time. We will assume that the thermal load on the outer surface of the heat-exchange device is a combination of the known heat flux density $q = q(t, x, z)$ and the interaction with the ambient medium, whose temperature is prescribed as a function of the coordinates and the time. All end surfaces of the device are adiabatically isolated, the root sections of the fins are in ideal thermal contact with the tube wall. We assume that the characteristic dimension of the device along the z coordinate is much larger than the characteristic transverse dimension along the x coordinate, and the change in temperature along the z axis is relatively small. The latter makes it possible to disregard the longitudinal component of the conductive heat flux compared to the transverse one. Let us assume that the influence of heat conduction on the temperature of the flow of the heat-transfer agent is small.

We consider the temperature distributions over all structural elements of the device and the distribution of the heat-transfer agent temperature along the z axis to be known at the initial time. At $t > 0$ the flow of the heat-transfer agent $G = G(t)$ with the prescribed temperature $\theta(t)$ is supplied to the inlet of the heat-exchange device ($z = 0$). The problem is to find the temperature distributions over the elements of the heat-exchange device at subsequent times. We note that in the described statement the model of the process a change in the coordinate within the limits from zero to infinity is possible. The latter is convenient when one needs to analyze the temperature field in the vicinity of the "inlet" to the heat-exchange apparatus.

In view of the assumptions made the mathematical formulation of the problem has the form

$$c_1\rho_1\delta_1 \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\lambda_1\delta_1 \frac{\partial u}{\partial x} + \alpha_1 (\omega_1 - u) + q_1 (t, x, z); \tag{4}$$

$$c_2\rho_2\delta_2 \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\lambda_2\delta_2 \frac{\partial v}{\partial x} + \alpha_2 (\omega_2 - v) + \widetilde{\alpha}_2 (\theta - v) + q_2 (t, x, z); \tag{5}$$

$$c_3\rho_3\delta_3 \frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\lambda_3\delta_3 \frac{\partial w}{\partial x} + \alpha_3 (\omega_3 - w) + \widetilde{\alpha}_3 (\theta - w) + q_3 (t, x, z); \tag{6}$$

$$c_4\rho_4\delta_4 \frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial x}\lambda_4\delta_4 \frac{\partial \vartheta}{\partial x} + \alpha_4 (\omega_4 - \vartheta) + q_4 (t, x, z); \tag{7}$$

$$c_p \rho F \frac{\partial \theta}{\partial t} + c_p G \frac{\partial \theta}{\partial z} = \int_0^{b_2} \widetilde{\alpha}_2 (v - \theta)\, dx + \int_0^{b_3} \widetilde{\alpha}_3 (w - \theta)\, dx; \tag{8}$$

$$u (0, x, z) = u^0 (x, z); \tag{9}$$

$$v (0, x, z) = v^0 (x, z); \tag{10}$$

$$w (0, x, z) = w^0 (x, z); \tag{11}$$

$$\vartheta (0, x, z) = \vartheta^0 (x, z); \tag{12}$$

$$\theta (0, z) = \theta^0 (z); \tag{13}$$

$$\frac{\partial u}{\partial x} (t, b_1, z) = 0, \quad t > 0; \tag{14}$$

$$\frac{\partial \vartheta}{\partial x} (t, 0, z) = 0, \quad t > 0; \tag{15}$$

$$u (t, 0, z) = v (t, 0, z) = w (t, 0, z), \quad t > 0; \tag{16}$$

$$\lambda_1\delta_1 \frac{\partial u}{\partial x} (t, 0, z) + \lambda_2\delta_2 \frac{\partial v}{\partial x} (t, 0, z) + \lambda_3\delta_3 \frac{\partial w}{\partial x} (t, 0, z) = 0, \quad t > 0; \tag{17}$$

$$v (t, b_2, z) = w (t, b_3, z) = \vartheta (t, b_4, z), \quad t > 0; \tag{18}$$

$$\lambda_2\delta_2 \frac{\partial v}{\partial x} (t, b_2, z) + \lambda_3\delta_3 \frac{\partial w}{\partial x} (t, b_3, z) + \lambda_4\delta_4 \frac{\partial \vartheta}{\partial x} (t, b_4, z) = 0; \tag{19}$$

$$\theta (0, t) = \theta^* (t), \quad t > 0. \tag{20}$$

We will apply the direct integral Laguerre transform in the variable t to problem (4) – (20). By using the vector-matrix form we obtain the following problem in the image space:

$$\frac{\partial}{\partial x}\lambda_1\delta_1\frac{\partial \mathbf{u}}{\partial x} - A_1\mathbf{u} = \mathbf{q}_1^* - \widetilde{A}_1\boldsymbol{\theta} \quad ; \tag{21}$$

$$\frac{\partial}{\partial x}\lambda_2\delta_2\frac{\partial \mathbf{v}}{\partial x} - A_2\mathbf{v} = \mathbf{q}_2^* - \widetilde{A}_2\boldsymbol{\theta} \quad ; \tag{22}$$

$$\frac{\partial}{\partial x}\lambda_3\delta_3\frac{\partial \mathbf{w}}{\partial x} - A_3\mathbf{w} = \mathbf{q}_3^* - \widetilde{A}_3\boldsymbol{\theta} \quad ; \tag{23}$$

$$\frac{\partial}{\partial x}\lambda_4\delta_4\frac{\partial \boldsymbol{\vartheta}}{\partial x} - A_2\boldsymbol{\vartheta} = \mathbf{q}_4^* - \widetilde{A}_4\boldsymbol{\theta} \quad ; \tag{24}$$

$$\frac{d\boldsymbol{\theta}}{dz} + A^*\boldsymbol{\theta} = \mathbf{f}^* + G^{-1}\left(\int_0^{b_2}\widetilde{A}_2\mathbf{v}dx + \int_0^{b_3}\widetilde{A}_3\mathbf{w}dx\right); \tag{25}$$

$$\frac{\partial \mathbf{u}}{\partial x}(b_1, z) = \mathbf{0}\,; \tag{26}$$

$$\frac{\partial \boldsymbol{\vartheta}}{\partial x}(0, z) = \mathbf{0}\,; \tag{27}$$

$$\mathbf{u}(0, z) = \mathbf{v}(0, z) = \mathbf{w}(0, z)\,; \tag{28}$$

$$\lambda_1\delta_1\frac{\partial \mathbf{u}}{\partial x}(0, z) + \lambda_2\delta_2\frac{\partial \mathbf{v}}{\partial x}(0, z) + \lambda_3\delta_3\frac{\partial \mathbf{w}}{\partial x}(0, z) = \mathbf{0}\,; \tag{29}$$

$$\mathbf{v}(b_2, z) = \mathbf{w}(b_3, z) = \boldsymbol{\vartheta}(b_4, z)\,; \tag{30}$$

$$\lambda_2\delta_2\frac{\partial \mathbf{v}}{\partial x}(b_2, z) + \lambda_3\delta_3\frac{\partial \mathbf{w}}{\partial x}(b_3, z) + \lambda_4\delta_4\frac{\partial \boldsymbol{\vartheta}}{\partial x}(b_4, z) = \mathbf{0}\,; \tag{31}$$

$$\boldsymbol{\theta}(0) = \boldsymbol{\theta}^*\,, \tag{32}$$

here

$$\boldsymbol{\theta}^* = \text{column}\left\{\theta_0^*, \theta_1^*, \theta_2^*\right\}, \ldots$$

where

$$\theta_k^* = \int_0^\infty \theta^* L_k(t)\exp(-t)\,dt, \quad k = 0, 1, 2, 3, \ldots;$$

$$(A_i)_{mn} = (\alpha_i L_m\,;\, L_n) + c_i\rho_i\delta_i D_{mn} + (\widetilde{A}_i)_{mn}\,;$$

$$(\widetilde{A}_i)_{mn} = \begin{cases} O_{mn}, & i = \overline{1, 4}, \\ (\alpha_i L_m ; L_n), & i = \overline{2, 3}; \end{cases}$$

$$(q_i^*)_n = (q_i^* ; L_n) + c\rho_i \delta_i (\overset{0}{u_i})_n ; \quad q_i^* = -(q_i + \alpha_i \omega_i), \quad i = \overline{1, 4};$$

$$(\overset{0}{u_1})_n = \overset{0}{u} ; \quad (\overset{0}{u_2})_n = \overset{0}{v} ; \quad \forall n ; \quad (\overset{0}{u_2})_n = \overset{0}{w} ; \quad (\overset{0}{u_4})_n = \vartheta^0 ; \quad \forall n ;$$

$$A^* = G^{-1} \left(\int_0^{b_2} \widetilde{A}_2 dx + \int_0^{b_3} \widetilde{A}_3 dx \right) + c_p \rho \, F G^{-1} D ; \quad G_{mn} = (c_p G L_m ; L_n) ;$$

$$\mathbf{f}^* = c_p \rho \, F G^{-1} \boldsymbol{\theta}^0 \; ; \quad \theta_n^0 = \theta^0 ; \quad \forall n ; \quad 0_n = 0 ; \quad \forall n ;$$

$$O_{mn} = 0 ; \quad \forall m ; \quad \forall n ; \quad n, m = 0, 1, 2, \ldots$$

We introduce the following notation:

$$\kappa = \text{column} \left\{ \mathbf{u}^{\mathrm{T}}, \mathbf{v}^{\mathrm{T}}, \mathbf{w}^{\mathrm{T}}, \boldsymbol{\vartheta}^{\mathrm{T}} \right\} ; \quad \kappa^* = \text{column} \left\{ (\mathbf{u}^*)^{\mathrm{T}}, (\mathbf{v}^*)^{\mathrm{T}}, (\mathbf{w}^*)^{\mathrm{T}}, (\boldsymbol{\vartheta}^*)^{\mathrm{T}} \right\},$$

where

$$\mathbf{u}^* = \lambda_1 \delta_1 \frac{\partial \mathbf{u}}{\partial x} ; \quad \mathbf{v}^* = \lambda_2 \delta_2 \frac{\partial \mathbf{v}}{\partial x} ; \quad \mathbf{w}^* = \lambda_3 \delta_3 \frac{\partial \mathbf{w}}{\partial x} ; \quad \boldsymbol{\vartheta}^* = \lambda_4 \delta_4 \frac{\partial \boldsymbol{\vartheta}}{\partial x} .$$

By rewriting systems of second-order equations (21)–(24) in an equivalent form we obtain for each system the following expression:

$$\frac{\partial}{\partial x} \begin{pmatrix} \kappa_i \\ \kappa_i^* \end{pmatrix} + R_i \begin{pmatrix} \kappa_i \\ \kappa_i^* \end{pmatrix} = \begin{pmatrix} 0 \\ \varphi_i \end{pmatrix}, \quad i = \overline{1, 4} . \tag{33}$$

The partitioned matrices R_i are defined by the relation:

$$R_i = \begin{pmatrix} O & -\dfrac{1}{\lambda_i \delta_i} E \\ -A_i & O \end{pmatrix}, \quad i = \overline{1, 4},$$

where E is a unit matrix ($Ea = a$);

$$\varphi_i = \mathbf{q}_i^* - \widetilde{A}_i \boldsymbol{\theta} \;, \quad i = \overline{1, 4} .$$

The general solution of Eq. (33) is as follows:

$$\begin{pmatrix} \kappa_i \\ \kappa_i^* \end{pmatrix} = \Phi^i(x, 0) \begin{pmatrix} \kappa_i(0) \\ \kappa_i^*(0) \end{pmatrix} + \int_0^x \Phi^i(x, \xi) \begin{pmatrix} \mathbf{0} \\ \varphi_i(\xi) \end{pmatrix} d\xi ,$$

where $\Phi^i(x, \xi)$ is a fundamental matrix of the system.

Under certain conditions

$$\Phi^i(x, \xi) = \exp \left[-\int_\xi^x R_i(\xi) \, d\xi \right] .$$

Taking account of the form of the vector in the integrand, it is convenient to write the general solution of Eq. (33) as

$$\kappa_i(x) = \Phi^i_{11}(x, 0)\,\kappa_i(0) + \Phi^i_{12}(x, 0)\,\kappa^*_i(0) + \int_0^x \Phi^i_{12}(x, \xi)\,\boldsymbol{\varphi}_i(\xi)\,d\xi ;$$
$$\kappa^*_i(x) = \Phi^i_{21}(x, 0)\,\kappa_i(0) + \Phi^i_{22}(x, 0)\,\kappa^*_i(0) + \int_0^x \Phi^i_{22}(x, \xi)\,\varphi_i(\xi)\,d\xi . \tag{34}$$

We note that $\boldsymbol{\theta}$ depends only on z; then relations (34) appear as

$$\kappa_i(x) = \Phi^i_{11}(x, 0)\,\kappa_i(0) + \Phi^i_{12}(x, 0)\,\kappa^*_i(0) + \mathrm{r}_i(x) + \widetilde{B}_i(x)\,\boldsymbol{\theta} ;$$
$$\kappa^*_i(x) = \Phi^i_{21}(x, 0)\,\kappa_i(0) + \Phi^i_{22}(x, 0)\,\kappa^*_i(0) + \mathrm{p}_i(x) + \widehat{B}_i(x)\,\boldsymbol{\theta} ; \tag{35}$$

where

$$\mathrm{r}_i(x) = \int_0^x \Phi^i_{12}(x, \xi)\,\mathrm{q}^*_i(\xi)\,d\xi , \quad i = \overline{1, 4} ;$$

$$\mathrm{p}_i(x) = \int_0^x \Phi^i_{22}(x, \xi)\,\mathrm{q}^*_i(\xi)\,d\xi , \quad i = \overline{1, 4} ;$$

$$\widetilde{B}_i(x) = \int_0^x \Phi^i_{12}(x, \xi)\,\widetilde{A}_i(\xi)\,d\xi , \quad i = \overline{1, 4} ;$$

$$\widehat{B}_i(x) = \int_0^x \Phi^i_{22}(x, \xi)\,\widetilde{A}_i(\xi)\,d\xi , \quad i = \overline{1, 4} .$$

We note that

$$\mathrm{r}_i(0) = \mathbf{0} ; \quad \mathrm{p}_i(0) = \mathbf{0} ; \quad i = \overline{1, 4} ; \quad \widetilde{B}_i(0) = O , \quad \widehat{B}_i(0) = O , \quad i = \overline{1, 4} .$$

It can easily be seen that the solution of the problem in the form (35) is completely determined if the vector $\boldsymbol{\theta}$ (we determine it below) and the components of the partitioned column vector g are known:

$$\mathrm{g} = \text{column}\left\{ \mathrm{u}(0)^{\mathrm{T}} , \ \mathrm{u}^*(0)^{\mathrm{T}} , \ \mathrm{v}(0)^{\mathrm{T}} , \ \mathrm{v}^*(0)^{\mathrm{T}} , \ \mathrm{w}(0)^{\mathrm{T}} , \ \mathrm{w}^*(0)^{\mathrm{T}} , \ \boldsymbol{\vartheta}(0)^{\mathrm{T}} , \ \boldsymbol{\vartheta}^*(0)^{\mathrm{T}} \right\} .$$

Requiring fulfillment of boundary conditions (26) – (31), we arrive at the following (partitioned) system of equations for determining the components of the vector g:

$$K\mathrm{g} = \mathrm{s}_0 + \boldsymbol{\lambda} , \tag{36}$$

where the nonzero components of the partitioned matrix K are determined by the relations

$$K_{11} = \Phi^1_{11}(b_1, 0) ; \quad K_{12} = \Phi^1_{22}(b_1, 0) ;$$

$$K_{28} = K_{31} = K_{41} = K_{52} = K_{54} = K_{56} = E ; \quad K_{33} = K_{45} = -E ;$$

$$K_{63} = K_{73} = \Phi^2_{11}(b_2, 0) ; \quad K_{64} = K_{74} = \Phi^2_{12}(b_2, 0) ; \quad K_{65} = -\Phi^3_{11}(b_3, 0) ;$$

$$K_{66} = -\Phi^3_{12}(b_3, 0); \quad K_{77} = -\Phi^4_{11}(b_4, 0); \quad K_{78} = -\Phi^4_{12}(b_4, 0);$$

$$K_{83} = \Phi^2_{21}(b_2, 0); \quad K_{84} = \Phi^2_{22}(b_2, 0); \quad K_{85} = \Phi^3_{21}(b_3, 0);$$

$$K_{86} = \Phi^3_{22}(b_3, 0); \quad K_{87} = \Phi^4_{21}(b_4, 0); \quad K_{88} = \Phi^4_{22}(b_4, 0).$$

The components of the partitioned column vector of the right-hand sides of Eq. (36) are

$$\mathrm{s}_0 = \text{column}\left\{(\mathrm{s}^1_0)^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, (\mathrm{s}^6_0)^{\mathrm{T}}, (\mathrm{s}^7_0)^{\mathrm{T}}, (\mathrm{s}^8_0)^{\mathrm{T}}\right\};$$

$$\lambda = \text{column}\left\{\lambda^{\mathrm{T}}_1, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \mathbf{0}^{\mathrm{T}}, \lambda^{\mathrm{T}}_6, \lambda^{\mathrm{T}}_7, \lambda^{\mathrm{T}}_8\right\}.$$

Here we adopt the notation

$$\mathrm{s}^1_0 = -\mathrm{p}_1(b_1); \quad \mathrm{s}^6_0 = \mathrm{r}_3(b_3) - \mathrm{r}_2(b_2); \quad \mathrm{s}^7_0 = \mathrm{r}_4(b_4) - \mathrm{r}_2(b_2);$$

$$\mathrm{s}^8_0 = -\mathrm{p}_2(b_2) - \mathrm{p}_3(b_3) - \mathrm{p}_4(b_4); \quad \lambda_1 = \widehat{B}_1(b_1)\boldsymbol{\theta} = M^1_*\boldsymbol{\theta};$$

$$\lambda_6 = [\widetilde{B}_2(b_2) - \widetilde{B}_3(b_3)]\boldsymbol{\theta} = M^6_*\boldsymbol{\theta}; \quad \lambda_7 = [\widetilde{B}_2(b_2) - \widetilde{B}_4(b_4)]\boldsymbol{\theta} = M^7_*\boldsymbol{\theta};$$

$$\lambda_8 = [\widehat{B}_2(b_2) + \widehat{B}_3(b_3) + \widehat{B}_4(b_4)]\boldsymbol{\theta} = M^8_*\boldsymbol{\theta}.$$

Let $W = K^{-1}$; then the components of the vector g are determined by the relation

$$\mathrm{g} = W\mathrm{s}_0 + W\lambda,$$

from which it follows that all the components of the partitioned vector g depend on θ in a linear manner.

We introduce the following notation:

$$\mathrm{g}^i_0 + W_{ij}\mathrm{s}^j_0; \quad M^i = W_{ij}M^j_*, \quad i, j = \overline{1, 8},$$

where s^j_0 is the j-th element of the partitioned vector s_0; W_{ij} is a square matrix, being the (ij)-th element of the partitioned matrix W.

By substituting the found values of the components of the vector g into expressions (35) we obtain

$$\kappa_i(x) = \Phi^i_{11}(x, 0)\,\mathrm{g}^{2i-1}_0 + \Phi^i_{12}(x, 0)\,\mathrm{g}^{2i}_0 + \mathrm{r}_i(x) + N^i\boldsymbol{\theta}, \tag{37}$$

where

$$N^i = \Phi^i_{11}(x, 0)\,M^{2i-1} + \Phi^i_{12}(x, 0)\,M^{2i} - \widetilde{B}_i(x), \quad i = \overline{1, 4}.$$

After substitution of relations (37) into Eq. (25) we find

$$\frac{d\boldsymbol{\theta}}{dz} + A\boldsymbol{\theta} + \mathrm{h}, \tag{38}$$

where

$$A = A^* - G^{-1}\left(\int_0^{b_2} \widetilde{A}_2 N^2 dx + \int_0^{b_3} \widetilde{A}_3 N^3 dx\right);$$

TABLE 1. Results of the Calculation

Temperature	t		
	0.1	0.5	1.0
	$z=0.1;\ x=0.04$		
u	0.00576	0.0238	0.0466
v	0.00594	0.0241	0.04688
	$z=0.1;\ x=0.08$		
u	0.00550	0.0237	0.0465
v	0.00589	0.0241	0.0468
	$z=0.4;\ x=0.08$		
u	0.00283	0.0210	0.0438
v	0.00320	0.0213	0.0440

$$\mathrm{h} = \mathrm{f}^* + G^{-1}\left\{ \int_0^{b_2} \tilde{A}_2\,[\Phi_{11}^2(x,\,0)\,\mathrm{g}_0^3 + \Phi_{12}^2(x,\,0)\,\mathrm{g}_0^4 + \mathrm{r}_2(x)]\,dx + \right.$$

$$\left. + \int_0^{b_3} \tilde{A}_3\,[\Phi_{11}^3(x,\,0)\,\mathrm{g}_0^5 + \Phi_{12}^3(x,\,0)\,\mathrm{g}_0^6 + \mathrm{r}_3(x)]\,dx \right\}.$$

We write out the solution of (38) with account for boundary condition (32):

$$\theta(z) = \Phi(z,\,0)\,\theta^* + \int_0^z \Phi(z,\,\xi)\,\mathrm{h}(\xi)\,d\xi\,,$$

where

$$\Phi(z,\,\xi) = \exp\left[-\int_\xi^z A(\xi)\,d\xi\right],$$

which completes the construction of the solution of the considered problem as a whole since all the sought functions are determined in the image space and the inverse transformation is performed by the rule (2).

The present work gives results of calculating temperature distributions over the fins, the tube walls, and the prescribed cross section of the flow of the heat transfer agent under the following assumptions: the initial temperature distributions over the element of the heat-exchange device (including the flow temperature) are uniform and are taken as the origin; a flow of the heat-transfer agent with dimensionless temperature equal to unity is supplied to the inlet of the element; there is no heat transfer with the ambient medium; the dimensionless length of the tube is $a = 1$; the dimensionless width of the fins is $b_1 = b_4 = 0.1$; the dimensionless value of the half-perimeter of the tube is $b_2 = b_3 = 0.1$ (the profile is symmetrric). The thermophysical properties of the material, the coefficients of heat transfer with the flow of the heat-transfer agent, and the flow rate and thermophysical properties of the heat-transfer agent are constant in time and space. The dimensionless flow velocity is 5.0, the dimensionless coefficient of heat transfer between the tube walls and the flow of the heat-transfer agent is 0.1, and between the flow of the heat-transfer agent and the tube walls it is 0.01. Two terms of the expansion are retained in the solution (for $L_0 = 1$ and $L_1 = 1 - t$), and in calculating the matrix exponent and the integrals by a recurrence method use is made of eight terms of the expansion.

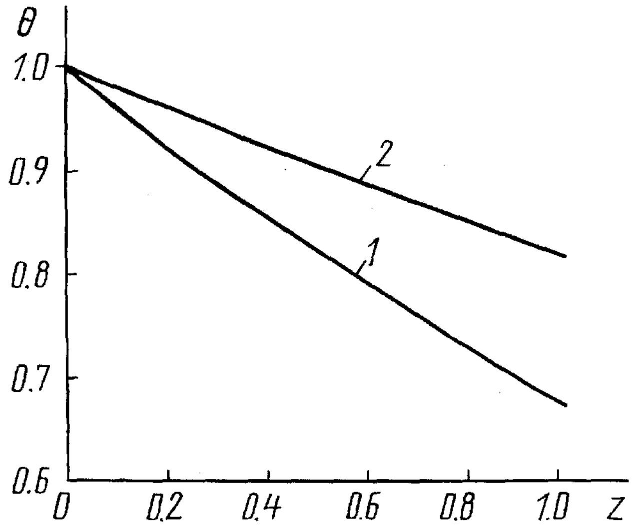

Fig. 2. Dimensionless temperature of the heat-transfer agent θ vs z coordinate at the time: 1) $t = 0.1$; 2) 1.0.

Results of the calculations are given in Table 1 and Fig. 2. It can easily be seen that even in the approximation adopted the solution agrees completely with the physical meaning of the problem. Table 1 takes account of the symmetry of the results of calculating the temperature in cross sections x = const that are located in the same manner relative to the axis of the tube cross section.

Compared to the method of the integral Laplace transform, use of the integral Laguerre transform in the time variable extends the possibility of investigating linear problems of conjugate heat transfer with variable parameters of the transfer.

NOTATION

u, ϑ, temperatures of the fins; v, w, temperatures of the tube walls; θ, temperature of the flow of the heat-transfer agent; α_i, $i = \overline{1, 4}$, coefficients of heat transfer from the ambient medium to the fins and the tube walls, respectively; ω_i, $i = \overline{1, 4}$, temperature distributions for the ambient medium; $\bar{\alpha}_i$, $i = \overline{2, 3}$, coefficients of heat transfer from the flow of the heat-transfer agent to the tube walls; q_i, density of the heat flux to the corresponding portions of the tube; c_i, λ_i, ρ_i, δ_i, $i = \overline{1, 4}$, heat capacity, thermal conductivity, density, and thickness of the fin and tube material; c_p, ρ, G, F, heat capacity, density, and flow rate of the heat-transfer agent, cross-sectional area of the tube; b_i, a, $i = \overline{1, 4}$, dimensions of the tube.

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